Centre of Mass

① $\vec{r}_{com} = \dfrac{\Sigma m\vec{r}}{\Sigma m}$ | $\vec{a}_{com} = \Sigma m\vec{a}/\Sigma m$ | $\vec{V}_{com} = \Sigma m\vec{v}/\Sigma v$

$\Rightarrow X_{com} = \dfrac{\Sigma mx}{\Sigma m}$, $Y_{com} = \dfrac{\Sigma my}{\Sigma m}$, $Z_{com} = \dfrac{\Sigma mz}{\Sigma m}$

② for two point masses

$m_1 \leftarrow r \rightarrow m_2$

$X_{com} = \dfrac{m_2 r}{m_1+m_2}$ (from m_1)

$X_{com} = \dfrac{m_1 r}{m_1+m_2}$ (from m_2)

③ for non point masses $\vec{r}_{com} = \dfrac{\int dm \cdot r}{\int dm}$

★ $dm = \lambda \, dl$

★ $dm = \sigma \, dA$ (mass is distri. over Area)

★ $dm = \rho \, dV$ (mass is distri. over volume)

④ com of some common system.

1) Rectangular plate (By symmetry)

$X_c = b/2$ | $Y_c = L/2$

2) A triangular plate

$Y_c = h/3$

3) semi-circular ring

$Y_c = \dfrac{2R}{\pi}$ | $X_c = 0$

4) A semi-circular ~~ring~~ disc

$Y_c = \dfrac{4R}{3\pi}$ | $X_c = 0$

5) hemispherical shell

$Y_c = R/2$ | $X_c = 0$

6) solid hemisphere.

$Y_c = 3R/8$ | $X_c = 0$

7) solid circular cone

$Y_c = h/4$ | $X_c = 0$

8) A circular cone (hollow)

$Y_c = h/3$ | $X_c = 0$

⑤ Cavity in object

$r_{rem} \cdot M_{rem} = r_{cutted} \cdot M_{cutted}$

$r_{cutted} \Rightarrow$ distance from com of main body to com of cutted body.

⑥ Motion of com.

$\vec{P}_{com} = \vec{P}_1 + \vec{P}_2 + \dots + \vec{P}_N$

$\vec{F}_{Net\,com} = \vec{F}_1 + \vec{F}_2 + \dots + \vec{F}_N$

⑦ Isolated system.

$\Rightarrow$ mutual Interaction forces

① $F_{net} = 0$ Internal forces get

$F_{com} = 0$ cancel out.

② $a_{com} = 0$

③ $V_{com} = U_{com}$ | $P_i = P_f$

$m_1\vec{u}_1 + m_2\vec{u}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$

$\Rightarrow$ momentum conserv^n holds good for isolated system. ($P_i = P_f$)

$M_1X_1 = M_2X_2$

$X_1, X_2 \Rightarrow$ dist. from com.

muzzle vel = relative vel

a) Gun – Bullet system & Man – planck system

$$V_{Recoil} = -\left[\frac{\Sigma mV}{M + \Sigma m}\right]$$

m & v direction important.

Σm = जिसको relative motion observe किया होगा।

M = Recoil होनेवाली body की mass.

– Internal forces are cancel out in both Gun – Bullet & man – planck system.

b) Man – Boat system

$$x_{Recoil} = -\left[\frac{\Sigma m x}{M + \Sigma m}\right]$$

$$X_{man} = X_{rel} + X_{recoil}$$