

$$\omega^2 R_e = 0.03$$

# Gravitation

$$g h_1 = g_2 h_2$$

$$① F = G \frac{m_1 m_2}{r^2}$$

$$g \propto \frac{1}{h}$$

② accel. due to gravity.

$$g = \frac{G M_e}{R_e^2} = \frac{4\pi^2}{3} \dots$$

$$G M_e = g R_e^2 \Rightarrow 8 \text{ km} \times R_e$$

③ Variation of 'g'.

① height.

$$g' = \frac{G M_e}{(R_e + h)^2} \text{ or } \frac{g R_e^2}{(R_e + h)^2}$$

when,  $h \ll R_e$

$$g' = g \left(1 - \frac{2h}{R_e}\right)$$

$h \gg R_e$

$$g' = \frac{g}{\left(1 + \frac{h}{R_e}\right)^2}$$

② depth

$$g' = g \left(1 - \frac{d}{R_e}\right) \text{ for all cases}$$

③ rotation of earth

$$g' = g - \omega^2 R_e \cos^2 \theta$$

( $\omega$  from equator)

④ orbital vel.

$$V_o = \sqrt{\frac{G M_e}{R_e + h}}$$

but not mass of any satellite (i.e. moon)

or close to earth surface

$$V_o = \sqrt{\frac{G M_e}{R_e}} \Rightarrow \sqrt{g R_e}$$

⑤ kinetic, potential & mechanical energy

$$k = \frac{G M_e m}{2r}$$

$$U = -\frac{G M_e m}{r}$$

$$E = -\frac{G M_e m}{2r}$$

$$k = |E| = \frac{|U|}{2}$$

⑥ work done in moving object from one orbit to other

$$W_{ag} = \pm \frac{G M_e m}{2r} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right]$$

⑦ work done in moving object from surface to

height 'h' above surface.

$$W_{ag} = mgh \times R \text{ when } h \gg R_e$$

$\frac{mgh}{(1 + \frac{h}{R_e})}$

⑧ work done in moving object from surface to circular orbit

$$W_{ag} = E_2 - E_1 = E_2 - U_1$$

$$\rho_e \propto R_e^3$$

$$\rho_e \propto R_e^3 \rightarrow \text{mean density}$$

$$T = 2\pi \sqrt{\frac{r^3}{GM}}$$

① escape vel!

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

$$v_e = \sqrt{2} v_o$$

$$8\sqrt{2} = 11.2 \text{ km/s}$$

\* Kepler's law

$$\frac{\Delta A}{\Delta t} = \frac{L}{2m} = \text{const}$$

M = mass of planet.

a) Escape vel. given to an object located at ht 'h'.

$$v_e = \sqrt{\frac{2GM_e}{(R_e + h)}}$$

② Time pd for elliptical orbit

$$T^2 = \frac{4\pi^2}{GM_\odot} a^3$$

major axis

T.pd for circular orbit

$$T^2 = \frac{4\pi^2}{GM_e} R^3$$

b) escape vel. for orbiting body.

$$v_e = \sqrt{\frac{GM_e}{R_e + h}}$$

③  $v_o$  of Geostationary orbit  $\Rightarrow 3.1 \text{ km/sec}$

$$(h = 6R_e)$$

④ escape vel. from surface to earth

$$v_e = \sqrt{\frac{2GM_e}{R_e}} = \sqrt{2gR_e}$$

$$\vec{I} = \frac{\vec{r} \times \vec{p}}{m}$$

$$\vec{E} = \vec{F}/q$$

$$\vec{I} = GM/v^2$$

$$\vec{E} = kq/r^2$$

$$I = -dv/dx$$

$$E = -dV/dx$$

modification

$$v_e = \sqrt{\frac{2GM_e}{R_e}}$$

a)  $v_{\text{given}} > v_{\text{escape}}$

$$v = (\sqrt{n^2 - 1}) \cdot v_e$$

$$n = \frac{v_{\text{given}}}{v_{\text{escape}}}$$

2nd Kepler's law

$$\frac{dA}{dt} = \frac{L}{2m}$$

angular momentum

$$L = I\omega$$

b)  $v_{\text{given}} < v_{\text{escape}}$

$$h = n^2 R_e \sqrt{1 - n^2}$$

from surface of earth

$L =$  angular momentum

earth - potential graphs are totally inverted from E, potential.

### Planetary motion

① Laplace = Laplace

(angular momentum is conserved)

$$m v_a \epsilon_a = m v_p \epsilon_p$$

$$\frac{v_a}{v_p} = \frac{\epsilon_p}{\epsilon_a} \Rightarrow \frac{v_{\min}}{v_{\max}} = \frac{\epsilon_p}{\epsilon_a} \Rightarrow \frac{a(1-e)}{a(1+e)} = \frac{(1+e)}{(1-e)}$$

shape of the orbit of a satellite

①  $v < v_o \rightarrow$  spiral path

$v = v_o \rightarrow$  circular path

$v > v_o \rightarrow$  elliptical path

$v = v_e$

$v = v_e \rightarrow$  parabolic path

$v > v_e \rightarrow$  hyperbolic path

② mechanical energy in elliptical orbit.

$$E = \frac{-GM_s M_p}{2a}$$

(a = semimajor axis)

③ speed at any point

$$v_{\max} = \sqrt{GM_s \left( \frac{2}{r_{\min}} - \frac{1}{a} \right)}$$