Projectile Motion

(A) ground to ground (θ)

(1) Range $\frac{2u_x \cdot u_y}{g} \Rightarrow \frac{u^2 \sin\theta}{g}$

(2) Max. ht $H = \frac{u_y^2}{2g}$ $\tan\theta = \frac{u_y}{u_x}$

(3) Time of flight $T = \frac{2u_y}{g}$

$\Rightarrow$ $R\tan\theta = 4H$

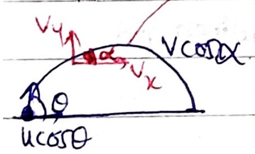
$\Rightarrow$ $H = \frac{1}{8} gT^2$

(4) velocity and displacement

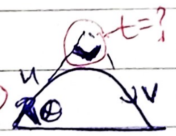
$\Rightarrow$ $V_x = u\cos\theta$

$\Rightarrow$ $V_y = u\sin\theta - gt$

$\pm\tan(\alpha) = \frac{V_y}{V_x} = \frac{u\sin\theta - gt}{V\cos\theta}$



$\therefore u\cos\theta = v\cos\alpha$

(5) 

$t = \frac{u}{g\sin\theta}$

(6) Displacement

$x = u\cos\theta\, t$ $y = u\sin\theta\, t - \frac{1}{2} gt^2$

Trajectory

(a) from posn – time graph

$y = x\tan\theta - \frac{1}{2} \frac{g x^2}{u^2\cos\theta}$

$y = ax - bx^2$

– Trajectory से R, H, U, θ, T.

$y = ax - bx^2$

$\Rightarrow$ $a \Rightarrow \tan\theta$

$R = x \Rightarrow a/b$

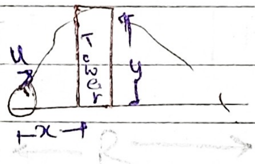
$H \Rightarrow \frac{a^2}{4b}$

$T = \sqrt{\frac{2a^2}{bg}}$

$R\tan\theta = 4H$

$H = \frac{1}{8} gT^2$

* special form of trajectory.



$Y = x\tan\theta \left[1 - \frac{x}{R}\right]$

* Same Range

(1) IF $\theta_1 + \theta_2 = 90°$

(2) $\theta_1 = \theta$, $\theta_2 = 90 - \theta$

(3) $\theta_1 = 45 - \theta$, $\theta_2 = 45 + \theta$

* Max. Range

(1) $R_{max} = \frac{u^2}{g}$ $(\theta = 45°)$

(2) $H = \frac{R_{max}}{4}$

Results based on same Range

(1) $H_1 + H_2 = \frac{R_{max}}{2}$

(2) $R = 4\sqrt{H_1 \cdot H_2}$

max. momentum

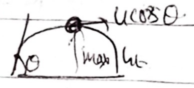
$\Delta P = P_f^{0} - P_i = m v^{0} - m v_0 \Rightarrow \boxed{-mgt}$

$\boxed{t = \frac{2u \sin\theta}{g}}$

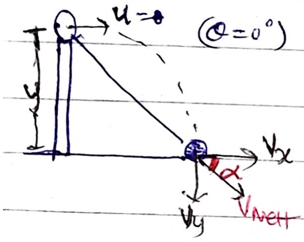
① Mechanical Energy (in projectile m.)

a) K.E at max. ht.

$$\boxed{K_{max} = K_i \cdot \cos^2\theta}$$



Object thrown from height



$$\boxed{x = ut}$$

$$\boxed{y = \frac{1}{2} g t^2}$$

$$\boxed{y = \frac{1}{2}\frac{g x^2}{u^2}}$$

$\Rightarrow \boxed{V_x = u}$ $\boxed{V_y = gt}$

$\Rightarrow \boxed{\tan\alpha = \frac{V_y}{V_x}}$

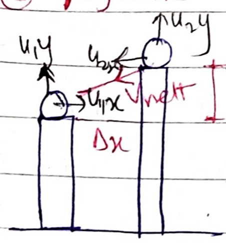
Components (Horizontal) are same.

$\boxed{u = V_{Net} \cos\alpha}$ $\boxed{u\cos\theta = v\cos\alpha}$ — in ground to ground.

② Hunter - Monkey problem

$$\boxed{t = \frac{d}{u}}$$

③ Projectile from diff. levels



$$\boxed{t_x = t_y}$$

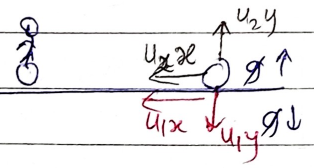
$$\boxed{\frac{\Delta x}{u_{1x} + u_{2x}} = \frac{\Delta y}{u_{1y} - u_{2y}}}$$

- time taken to travel along x-axis equals to time taken to travel along y-axis, then projectile will collide.

$\boxed{I_{new} = I_0 \cdot \sin^2\theta}$

↓ MOI of inclined rod

Relative Motion in projectile.



$a_{rel} = 0$
velocity = const
path = straight line

$\boxed{u_{1y} \Rightarrow u_{2y}}$ — necessary condⁿ

$$\boxed{t_{coll^n} \Rightarrow \frac{d}{u_{1x} + u_{2x}}}$$