Rotational Motion

① Moment of Inertia.

$\boxed{I = mr^2}$

(r is $\perp^{er}$ dist from axis of rotatn)

∴ 'I' is analogous to mass in Rotational.M.

② $\boxed{\tau_N = I\alpha}$ $\boxed{F_N = ma}$

$\boxed{K = \frac{1}{2} I\omega^2}$ $\boxed{K = \frac{1}{2} mv^2}$

a) Two point masses:

$\boxed{I_{com} = m_{red} \cdot r^2}$ ✗

$\boxed{m_{red} = \frac{m_1 m_2}{m_1 + m_2}}$ ★

b) Theorem of $\perp^{er}$ axis

ⓐ
$I_z = I_x + I_y$
$I_x = I_y + I_z$
$I_z = I_x + I_y$
} when 'z' coordinate not given.

ⓑ $\boxed{I_x = mr^2 = m(y^2 + z^2)}$ (when 'z'-coordinate given)

② MOI of non-point mass:

① Rod: $\boxed{I_{com} = \frac{1}{12} MR^2}$

② Ring: $\boxed{I_{com} = MR^2}$

③ Disc: $\boxed{I_{com} = \frac{1}{2} MR^2}$

④ Hollow cyl.: $\boxed{I_{com} = MR^2}$

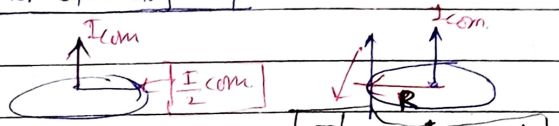
⑤ solid cylinder: $\boxed{I_{com} = \frac{1}{2} MR^2}$

⑥ Hollow sphere: $\boxed{I_{com} = \frac{2}{3} MR^2}$

⑦ solid sphere: $\boxed{I_{com} = \frac{2}{5} MR^2}$

③ Theorem of $\parallel^{el}$ to $\perp^{er}$ axis.

for symmetry only



$\boxed{I_{com}' = I_{com} + Mh^2}$

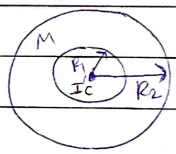
④ MoI about Inclined axis.

$\boxed{I_{inclined} = I_{axis} \cdot \sin^2\theta}$

⑤ Radius of Gyration

$\boxed{I_{com} = mK^2}$

⑥ cavity



$\boxed{I_c' = \frac{1}{2} M(R_1^2 + R_2^2)}$

⑦ Torque

$\vec{\tau} = \vec{r} \times \vec{F}$
$\tau = r \cdot F \cdot \sin\theta$
θ = angle b/w $\vec{r}$ & $\vec{F}$

⑧ Rotational equilibrium

① Translational eqm ($\tau \neq 0$)

② Rotational (eqm) ($F_{Net} \neq 0$)

③ static eqm → $\boxed{\tau_{clockwise} = \tau_{anticlockwise}}$

⑨ Work & Power.

① Translatory (motion) $\boxed{W = \int \vec{F} \cdot d\vec{r}}$

② Rotatory motion $\boxed{W = \int \vec{\tau} \, d\theta}$

⑩ Work - Energy theorem

$\boxed{\Sigma W = K_f - K_i = \frac{1}{2} I(\omega_f^2 - \omega_i^2)}$

only for rotatory

$$\boxed{f_s = \frac{\gamma mg \sin\theta}{(1+\gamma)}} \quad \boxed{\mu_s \geq \frac{\gamma \, mg \tan\theta}{1+\gamma}}$$

(11) Power :

$$\boxed{P = \frac{\Delta W}{time} = \frac{\Delta K}{time}} \quad \boxed{P_{inst} = \vec{\tau} \cdot \vec{\omega}}$$

(12) $\boxed{\tau = I\alpha = r \cdot F}$

(13) M.E.C in Rotation

$$\boxed{K_i + U_i = K_f + U_f}$$

(14) Translation & Rotation

(1) T.M & in it, each particle cover same distance, with same speed & same accl^n as that of com.

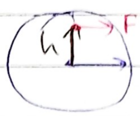
(2) R.M & $\boxed{a_t = r\alpha}$

Rolling

→ T.M + R.M = Rolling

(1) Pt. of Application of force,
(friction absent then for stop to body)

$$\boxed{h = \frac{I}{MR}}$$



(2) Rolling on Inclined plane

(1) $\boxed{I_{gole} = \gamma MR^2}$ $\boxed{\gamma_{cylinder\,(Hollow)} = \frac{1}{2} \ 1}$

$$\boxed{\gamma = \frac{K^2}{R^2}}$$

(2) static friction changes 'ω' not 'V'.

$$\boxed{a_{Roll} = \frac{g \sin\theta}{(1+\gamma)}} \quad \boxed{a_{slipping} = g \sin\theta}$$

in kinematics

→ min. coeff. of friction

$f_s \leq f_{s\,max} \Rightarrow \boxed{\mu_s \geq f_s / N}$

must use for rotational motion

(3) $V = \sqrt{\frac{2 g \sin\theta}{(1+\gamma)} \, l}$

(4) K.E of Rolling body at inclined plane.

→ for rolling motion

$$\boxed{K.E_{total} = \frac{1}{2} mv^2 (1+\gamma)}$$

K.E (translat^n) = $\frac{1}{2} mv^2$

K.E (rotat^n) = $\gamma \frac{1}{2} mv^2$

Angular momentum (L) for

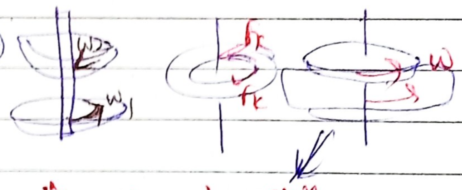
(1) $L_{orbital} \rightarrow mv \, r \sin\theta$ Rotation

(2) $L_{spin} \rightarrow I_{spin} \cdot \omega$ Rotation

conservation of angular momentum.

$$\boxed{\frac{dL}{dt} = 0}$$

(5)



like Inelastic colli^n.

$$\boxed{\omega_{common} = \frac{I_1\omega_1 \pm I_2\omega_2}{I_1 + I_2}}$$

$$\boxed{\Delta K = \frac{1}{2} \frac{I_1 I_2}{(I_1 + I_2)} (\omega_1 \mp \omega_2)^2}$$

Heat of capacitor

$$\boxed{\Delta H = \frac{1}{2} \frac{C_1 C_2}{(C_1 + C_2)} (V_1 \mp V_2)^2}$$